

## THE SIX-PHOTON AMPLITUDES

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Thanks to the absence of tree order, the six-photon processes is a good laboratory to study multi-leg one-loop diagrams. Particularly, there are enough on-shell external legs to observe a special Landau singularity: the double parton scattering.

### 1 Introduction

#### 1.1 Motivations

At LHC, we hope to discover new physics by the collisions between two protons. The partonic processes constitute a background, which is mandatory to know, if we want to observe new particles. In QCD, the coupling constant depends on an unphysical energy scale and to reduce this dependency, we have to increase the order of the expansion. So, new efficient methods, based on unitarity, have been developed<sup>1,2</sup>, for the NLO (Next to Leading Order) calculation. As the six-photon amplitudes have no rational terms, and no divergences therefore they are a good laboratory to apply those methods to multi-leg one-loop diagrams.

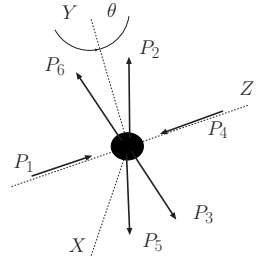
#### 1.2 Difficulties of NLO calculation

The amplitude of a six-photon diagram is the product of two terms, a tensor with the polarisation vectors of the external photons and a tensor integral. The first difficulty is to find a clever formulation of the polarisation vectors to simplify the expression and to obtain a compact result, and the second is to reduce efficiently the tensors integrals. The two solutions are the spinor formalism with helicity amplitudes, described in<sup>3</sup> and the efficient reduction thanks to unitarity-cuts<sup>1,2</sup>.

## 2 Results and Plots

In the past, three teams have calculated the six photons amplitudes analytically or numerically in QED<sup>4,5,6</sup>. I obtain very compact expressions for all the six-photon helicity amplitudes in QED, scalar QED and supersymmetric QED<sup>N=1</sup><sup>7</sup>. Each amplitude is a linear combination of four-point scalar integrals in  $n + 2$  dimensions and three point three-external-mass scalar integrals in  $n$  dimensions.

Let us plot the amplitudes in the Nagy-Soper kinematical configuration<sup>4</sup>. The photons 1 and 4 constitute the initial state along the z-axis whereas the photons 2, 3, 5 and 6 the final state. In the center of mass frame of the initial state, we put the final state at the phase space point:



$$\begin{cases} \vec{p}_2 = (-33.5, -15.9, -25.0) \\ \vec{p}_3 = (11.0, 13.2, 22.0) \\ \vec{p}_5 = (12.5, -15.3, -0.3) \\ \vec{p}_6 = (10.0, 18.0, 3.3) \end{cases} \quad (1)$$

New kinematical configuration are generated, by rotating the final state by an angle  $\theta$  about the y-axis, perpendicular to the z-axis. In the figure 1, we plot the NMHV (Next to Maximal Helicity Violating) amplitudes versus  $\theta$ . Two peaks appear for each amplitude at the angles

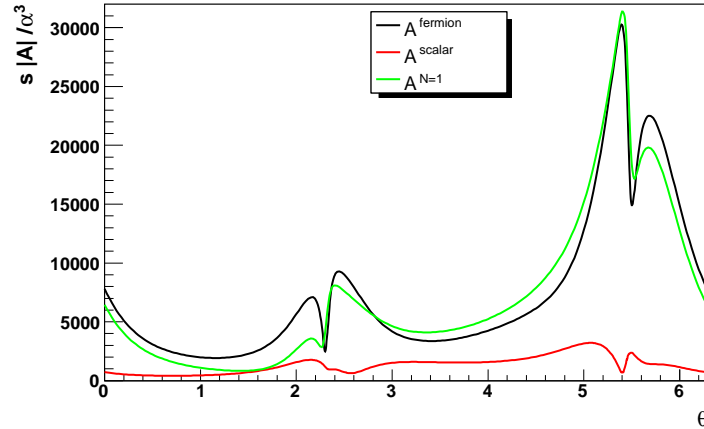


Figure 1: NMHV helicity amplitude of the six-photon process

$\theta_1 \simeq 2.32$  and  $\theta_2 \simeq \pi + 2.32 \sim 5.46$ . To understand the origins of these peaks, we split the final state in two photon pairs (3,5) and (2,6). We note  $k_t$  the transverse momentum of each photon pair and we plot its value versus the angle  $\theta$  on the left-graph of the figure 2. The peaks occurs exactly at the points where  $k_t$  is the smallest : it is the signature of double parton scattering. It is a special kinematical configuration, corresponding to a Landau singularity.

## 3 Landau singularity

Physically, Landau singularities correspond to a "resonance" of the virtual loop-particle with a physical process. For the six-photon amplitudes, this physical process is represented by diagrams on the figure 2. The two ingoing photons 1 and 4 split each into a fermion anti-fermion collinear

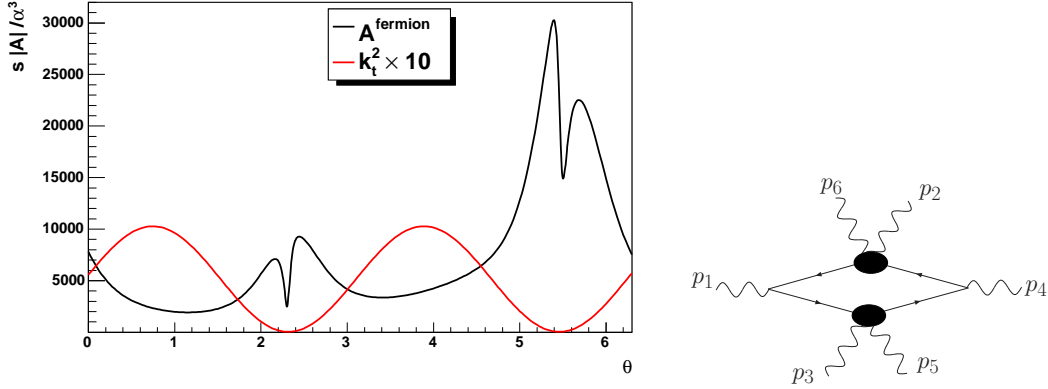


Figure 2: Localisation and kinematical configuration of double partons scattering

pair, then each fermion scatters with the anti-fermion to give a photon pair with no transverse momentum.

In a one-loop diagram, a Landau singularity is defined by finite points in the phase space, where the integrand of the loop is not analytic. But even if, locally the denominator is zero for example, the integral may be finite. We want to know if there are some divergences in the special case of this processes.

We reach the singularity when the transverse momentum of each pair of photons is equal to zero. With the Nagy-Soper kinematical configuration, we cannot reach it, so we modify it. As we rotate around the y-axis, we add or subtract a y-momentum  $\Delta^y$  for each final photons, to keep on the energy-momentum conservation :

$$\begin{cases} \vec{k}_3 = (33.5, 15.9 - \Delta^y, 25.0) \\ \vec{k}_4 = (-12, 5, 15.3 + \Delta^y, 0.3) \\ \vec{k}_5 = (-10.0, -18.0 + \Delta^y, -3.3) \\ \vec{k}_6 = (-11.0, -13.2 - \Delta^y, -22.0) \end{cases} \quad (2)$$

$\Delta^y$  acts as a regulator and the singularity is reached at  $\Delta^y = 1.05$ . Let us plot the QED amplitude around the singularity for several values of this regulator: The amplitude behaves as

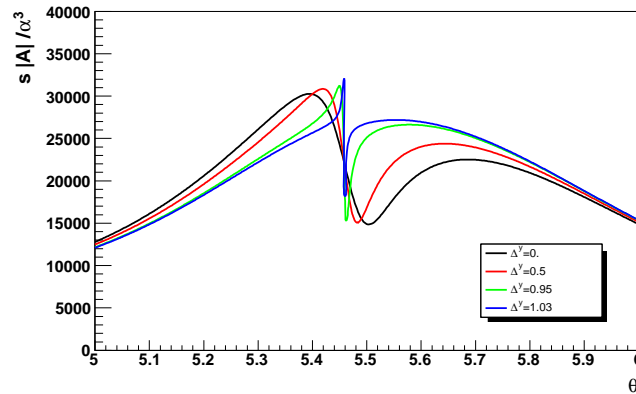


Figure 3: Around the double partons scattering

a wave around the singularity. It is larger for  $\Delta^y = 0$  and disappears completely for  $\Delta^y = 1.05$ .

The closer  $\Delta^y$  is from 1.05, the more squeezed the structure support is. There is no divergence because the numerator of the six-photon amplitudes vanish at the Landau singularity fast enough to regularize it. More explanations are given in<sup>8</sup>.

### 3.1 Summary

The six-photon amplitude is a good laboratory to study one-loop multi-legs diagrams, particularly the "analyticity" of the integrand of the loop. The non-analytic phase space points, called Landau singularities, let traces when plotting the amplitude (the double parton scattering). Fortunately, the structure of QED regularize them.

## References

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